MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

PCL0016 Calculus

(FOUNDATION ENGINEERING)

5 March 2018
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTION TO STUDENT

1. This examination paper consists of **4 pages** (including the cover page) with **6 questions only.**

2. Questions **1** and **6** are allocated with **20** marks while Questions **2, 3, 4** and **5** are allocated with **15** marks. **Attempt ALL questions.**

3. Please write all your answers in the Answer Booklet provided. **Show all relevant steps** to obtain maximum marks.

Question 1 [20 marks]

(a) A function $f(x)$ is denoted as follows

$$f(x)=\begin{cases} x^3\cos\dfrac{1}{x}, & x\neq 0 \\ \dfrac{}{x^2+1}, & \\ 0, & x=0 \end{cases}.$$

(i) Using Squeeze Theorem, find $\lim_{x\to 0} f(x)$. [4 marks]

(ii) Show that $f(x)$ is continuous at $x = 0$. [2 marks]

(b) Evaluate the following limits, if they exist

(i) $\lim_{x\to-\infty}\dfrac{x^3}{\sqrt{x^6-4}}$ [2 marks]

(ii) $\lim_{x\to-\infty}\dfrac{5x^{12}+3x^{10}-2}{3x^{12}+6x^{11}+10x^2-2x+1}$ [2 marks]

(iii) $\lim_{x\to\infty}\dfrac{x^3+x^2+x+1}{e^{3x}}$ (Hint: Use L'Hôpital's rule) [2 marks]

(iv) $\lim_{x\to 0}\dfrac{\tan 3x}{x^2}$ [3 marks]

(c) A nuclear scientist was working on an experiment and he had found a function $f(x)$ that represents the number of molecules of a radioactive material as denoted as follows

$$f(x)=\frac{ax^2-b}{x-3}$$

where x is the time in minutes since start of the reaction. In this experiment, he found out that the limit of the function as x approaches 3 is 6. Find the values of a and b. [5 marks]

Question 2 [15 marks]

(a) (i) Determine an expression for the slope of a tangent line to the curve $y=\cos x$ [1 mark]

(ii) At which points on the given curve are the tangent lines (found in (a)(i)) perpendicular to $y=2x+3$? [3 marks]

Continued...

(b) Find $\frac{dy}{dx}$ for the following functions:

(i) $y = \ln(\sin x)^{\tan(x)}$ [3 marks]

(ii) $x^2 \sin y + 2y^2 = e^{2x} + 3x^2$ [4 marks]

(iii) $y = \frac{x^3 \sqrt[3]{x^2 + 2x}}{(x^3 + 1)^4}$ [4 marks]

Question 3 [15 marks]

(a) As illustrated in Figure 3.1, a sheet of cardboard 6 feet by 8 feet will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions (width, length and height) of the box with the largest volume? [5 marks]

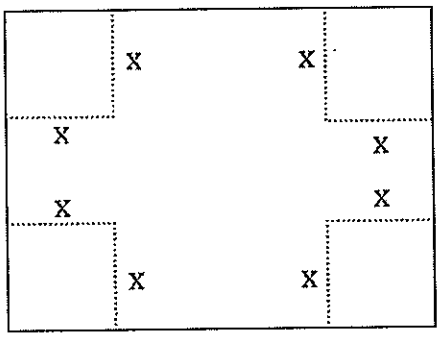

Figure 3.1

(b) A function $f(x)$ is denoted by $f(x) = x^4 - 4x^3 + 4x^2 - 12$.

(i) Find the interval(s) on which $f(x)$ is increasing and/or decreasing. [4 marks]

(ii) Find the interval(s) on which $f(x)$ is concave up and/or down. [3 marks]

(c) A particle moves in a straight line according to this equation of motion $s(t) = t^3 - 3t^2 - 5t + 12$ where s is measured in meter and t is measured in second. Is the particle speeding up or slowing down at $t = 2$s? [3 marks]

Continued...

Question 4 [15 marks]

(a) Given $f(x)=\dfrac{x^2}{(2x+1)(x+2)^2}$. Evaluate $\int f(x)dx$ using partial fractions.

[6 marks]

(b) Evaluate the following integrals:

(i) $\int \dfrac{\sin(x)\cos(x)}{1+\sin^2(x)}dx$ (Hint: Use integration by subsititution) [3 marks]

(ii) $\int x^2 \ln|x|\,dx$ (Hint: Use integration by parts) [3 marks]

(iii) $\int_{-4}^{2} f(x)dx$ if $f(x)=\begin{cases}\dfrac{6}{x} & \text{if } x>1\\ e^{4x} & \text{if } x\le 1\end{cases}$ [3 marks]

Question 5 [15 marks]

(a) Sketch and shade the area bounded by the curves $y=x^2$, $y=2-x$ and $y=1$. Label all the intersection points and find the area of the shaded region. [6 marks]

(b) Determine the volume of the solid obtained by rotating the region bounded by $y=x^2$ and $y=\sqrt{x}$ about the line $y=4$. [4 marks]

(c) Find the arc length of $f(x)=\dfrac{x^2}{8}-\ln x$ from $(1, f(1))$ to $(2, f(2))$. [5 marks]

Question 6 [20 marks]

(a) Form a differential equation for the following functions by eliminating arbitrary constant(s):

(i) $y^2=4ax$ [3 marks]

(ii) $y=a\cos(\ln x)+b\sin(\ln x)$ [3 marks]

(b) Solve the following differential equations:

(i) $y''+4y'+4y=0,\ y(0)=1,\ y'(0)=1$ [6 marks]

(ii) $\dfrac{dy}{dx}=\dfrac{x^2+y^2}{2xy}$. (Hint: Use $y=vx$) [8 marks]

End of Paper